



Β' ΛΥΚΕΙΟΥ
ΑΛΓΕΒΡΑ
ΓΕΝΙΚΗΣ ΠΑΙΔΕΙΑΣ

ΑΠΑΝΤΗΣΕΙΣ

ΘΕΜΑ 10

A. θεωρία σχολικό βιβλίο σελ. 28

B. iii

Γ. α)Λ β)Σ γ)Λ δ)Σ ε)Λ

Δ. α. $\sin(\alpha - \beta) = \sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta$

β. $\log_e \cdot \ln 10 = \frac{\ln e}{\ln 10} \cdot \ln 10 = 1$

γ. $\log \frac{\theta_1}{\theta_2} = \log \theta_1 - \log \theta_2$

ΘΕΜΑ 20

A. Πρέπει : $x^2 = 1 \cdot (2 - x) \Leftrightarrow x^2 = 2 - x \Leftrightarrow x^2 + x - 2 = 0$

$\Delta = 1 + 8 = 9$

$x = \frac{-1 \pm 3}{2} = \begin{cases} 1 \\ -2 \end{cases}$

B. Πρέπει : $\left. \begin{array}{l} P(1) = 0 \\ P(-2) = 0 \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} 1^4 + (\alpha - \beta) \cdot 1^3 - (2\alpha - 3\beta) \cdot 1^2 + 1 - 2 = 0 \\ (-2)^4 + (\alpha - \beta)(-2)^3 - (2\alpha - 3\beta)(-2)^2 + (-2) - 2 = 0 \end{array} \right\}$

$\Leftrightarrow \left\{ \begin{array}{l} 1 + \alpha - \beta - 2\alpha + 3\beta + 1 - 2 = 0 \\ 16 - 8\alpha + 8\beta - 8\alpha + 12\beta - 2 - 2 = 0 \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} -\alpha + 2\beta = 0 \\ -16\alpha + 20\beta = -12 \end{array} \right\} \Leftrightarrow$

$\left. \begin{array}{l} -\alpha + 2\beta = 0 \\ -4\alpha + 5\beta = -3 \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} \alpha = 2\beta \\ -8\beta + 5\beta = -3 \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} \alpha = 2 \\ \beta = 1 \end{array} \right.$

ΘΕΜΑ 30

A. Πρέπει : $\alpha_2 - \alpha_1 = \alpha_3 - \alpha_2 \Leftrightarrow \frac{1}{2} \sin 2\alpha - 1 = -2\eta\mu^2\alpha - \frac{1}{2} \sin 2\alpha \Leftrightarrow \sin 2\alpha = 1 - 2\eta\mu^2\alpha$ (ισχύει)

B. $\omega = \alpha_2 - \alpha_1 = \frac{1}{2} \sin 2\alpha - 1 = \frac{1 - 2\eta\mu^2\alpha}{2} - 1 = -\eta\mu^2\alpha - \frac{1}{2}$

$$S_4 = -2 \Leftrightarrow \frac{4}{2} \left[2 \cdot 1 + (4-1) \cdot \left(-\eta\mu^2\alpha - \frac{1}{2} \right) \right] = -2 \Leftrightarrow 4 - 6\eta\mu^2\alpha - 3 = -2$$

$$6\eta\mu^2\alpha = 3 \Leftrightarrow \eta\mu^2\alpha = \frac{1}{2} \begin{cases} \eta\mu\alpha = \frac{\sqrt{2}}{2} \\ \eta\mu\alpha = -\frac{\sqrt{2}}{2} \end{cases} \quad \begin{matrix} \eta\mu\alpha = \eta\mu\frac{\pi}{4} \\ \alpha = 2\kappa\pi + \frac{\pi}{4} \end{matrix} \quad \eta\alpha = 2\kappa\pi + \frac{3\pi}{4} \quad \eta$$

$$\eta\mu\alpha = \eta\mu\left(-\frac{\pi}{4}\right), \quad \kappa \in \mathbb{Z}$$

$$\alpha = 2\kappa\pi - \frac{\pi}{4} \quad \eta \quad \alpha = 2\kappa\pi + \frac{5\pi}{4}$$

Γ. $S_{103} = \frac{103}{2} \left[2 \cdot 1 + (103-1) \cdot \left(-\eta\mu^2\frac{\pi}{4} - \frac{1}{2} \right) \right] = 103[1 + 51 \cdot (-1)] = 103 \cdot (-50) = -5150$

Δ. $S_5 = \frac{5}{2} \left[2 \cdot 1 + (5-1) \cdot \left(-\eta\mu^2\alpha - \frac{1}{2} \right) \right] = 5[1 - 2\eta\mu^2\alpha - 1] = -10\eta\mu^2\alpha \neq 0, \quad \alpha \in \left(0, \frac{\pi}{2}\right)$

(Αν $S_5 = 0 \Leftrightarrow \dots \eta\mu^2\alpha = 0$ Άτοπο)

Άρα ο βαθμός του πολυωνύμου είναι 5.

ΘΕΜΑ 40

A. Πρέπει $2e^{2x+1} + e^{x+1} > 0$ Άρα $x \in \mathbb{R}$ δηλαδή $A_f = \mathbb{R}$

$$f(0) = \ln(2e^{2 \cdot 0 + 1} + e^{0+1}) = \ln(2e + e) = \ln 3e = \ln 3 + \ln e = \ln 3 + 1$$

B. $f(x) = 1 \Leftrightarrow \ln(2e^{2x+1} + e^{x+1}) = \ln e \xrightarrow{\ln x: 1-1} 2e^{2x+1} + e^{x+1} = e$

$$2e \cdot e^{2x} + e \cdot e^x - e = 0 \Leftrightarrow e(2e^{2x} + e^x - 1) = 0 \Leftrightarrow \left. \begin{matrix} 2(e^x)^2 + e^x - 1 = 0 \\ \text{θετω } e^x = y \end{matrix} \right\} \Rightarrow$$

$$2y^2 + y - 1 = 0 \cdot \begin{cases} y = -1 \\ \eta \\ y = \frac{1}{2} \end{cases}$$

Άρα $e^x = -1$ ΑΔΥΝΑΤΗ ή $e^x = \frac{1}{2} \Leftrightarrow \ln e^x = \ln \frac{1}{2} \Leftrightarrow x \cdot \ln e = \ln 1 - \ln 2 \Leftrightarrow x = -\ln 2$

Γ. $f(x) < 1 \Leftrightarrow \dots 2e^{2x} + e^x - 1 < 0, \quad -1 < e^x < \frac{1}{2}, \quad e^x > -1 \Leftrightarrow x \in \mathbb{R}$

$$e^x < \frac{1}{2} \xrightarrow{\ln x \uparrow} \ln e^x < \ln \frac{1}{2} \Leftrightarrow x < -\ln 2$$

Άρα $x \in (-\infty, -\ln 2)$